Polyhedral Analysis of Cardinality Constrained Combinatorial Optimization Problems

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Given a combinatorial optimization problem and a subset N of natural numbers, we obtain a cardinality constrained version of this problem by permitting only those feasible solutions whose cardinalities are elements of N. The cardinality constraint given by N, can also be written in form of a so called *cardinality sequence* $c = (c_1, \dots, c_m)$, where $c_1 < c_2 < \cdots < c_m$ and $\{c_1, \dots, c_m\} = N$. Well-known examples of cardinality constrained combinatorial optimization problems are the traveling salesman problem and the minimum odd cycle problem. Both problems are for itself combinatorial optimization problems, but in the line of sight of the minimum cycle problem, they are cardinality restricted versions of the latter problem.

Let Π be a combinatorial optimization problem defined on a finite set E and

$$\begin{array}{ll} \max & w^T x \\ \text{s.t.} & Ax \le b \\ & x_e \in \{0,1\} \;\; \forall\, e \in E \end{array} \tag{1}$$

an integer programming formulation of Π. Then, by adding the *cardinality bound* $c_1 \le x(E) \le c_m$ and Grötschel's[1] *cardinality forcing inequalities*

$$\begin{array}{l} (c_{p+1} - |F|)z(F) \; - \; (|F| - c_p)z(E \setminus F) \le c_p(c_{p+1} - |F|) \\ \quad \text{for all } F \subseteq E \text{ with } c_p < |F| < c_{p+1} \text{ for some } p \in \{1, \dots, m-1\}, \end{array} \tag{2}$$

we obtain an IP-formulation of the cardinality constrained version of Π. Since inequalities (2) are in general not facet defining for the associated polyhedra – at least in this form –, we will present one approach to strengthen them.

[1] M. Grötschel, *Cardinality homogeneous set systems, cycles in matroids, and associated polytopes*, 2004.