A simple max-cut algorithm for planar graphs

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The MAX-CUT problem asks for partitioning the node set V of a (weighted) graph $G = (V, E)$ into two sets (one of which might be empty), such that the sum of weights of edges joining nodes in different partitions is maximum. Cut problems have many applications, e.g. in VIA minimization in the layout of electronic circuits, in physics of disordered systems, or in network reliability. Furthermore, the problem is equivalent to unconstrained quadratic 0-1 optimization.

For general edge weights, the MAX-CUT problem (and by inversing the signum of the edge weights also the MIN-CUT problem) is NP-hard. However, it is polynomially solvable for certain classes of graphs. For planar graphs, for example, there exist several polynomial-time methods determining maximum (minimum) cuts for arbitrary choice of edge weights. In this case, the problem is typically solved by computing a minimum-weight perfect matching in some associated graph. In this work, we present a new and simple algorithm for determining maximum (minimum) cuts for arbitrary weighted planar graphs. Its running time can be bounded by $O(|V|^{\frac{3}{2}} \log |V|)$, similar to the fastest known method introduced by Shih, Wu and Kuo. However, our transformation yields a much smaller associated graph. Furthermore, it can be computed fast. As the practical running time strongly depends on the size of the associated graph, it can be expected that our algorithm is considerably faster than the methods known in the literature. More specifically, our program can determine maximum (minimum) cuts in huge realistic and random planar graphs with up to 10^6 nodes.

To this end we construct the dual graph which is expanded using K_4 subgraphs (so-called Kasteleyn cities – inspired by a work of Kastelyn cities in the 1960s) such that matchings in the latter yield Eulerian subgraphs of the dual. It is well known that there exists a one-to-one correspondence between Eulerian subgraphs in the dual and cuts in the primal graph. Thus, a minimum-weight perfect matching in the transformed graph yields an optimum cut in the primal graph. The transformation can be done in linear time. Together with the planar separator theorem by Lipton and Tarjan this leads to an algorithm with the same asymptotic running time as the one of Shih, Wu, and Kuo. However, in our transformation the expanded graph has a simpler structure and contains a considerably smaller number of both nodes and edges. As the bulk of the running time is spent in the matching computation and the latter scales with the size of the graph, our algorithm will be much faster in practice.