Inverse Tension Problems and Monotropic Optimization

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For optimization problems with estimated problem parameters one often knows a priori an optimal solution based on observations or experiments, but is interested in finding a set of parameters, such that the known solution is optimum (a) and the deviation from the initial estimates is minimized (b). The problem of recalculating the parameters satisfying (a) and (b) is known as *inverse optimization problem.* Among several inverse optimization problems the inverse network flows have been intensely investigated. As opposed to flows, *tension problems*, which are duals of flow problems, and their inverse versions have vastly been neglected. Our aim in this study is to fill this blank of the literature and extend the results of Ahuja and Orlin [1] for tensions. Moreover, we exploit monotropic programming (Rockafellar [2]) with linear cost functions to generalize these combinatorial results for a larger set of inverse problems including the inverse problems of generalized network flows.

Let $G = (N, A)$ be a connected digraph with node set N containing n nodes and arc set A containing m arcs, and a_{ij} represent an arc with tail node i and head node j. A vector $\theta \in \mathbb{R}^A$ is a *tension* on graph G with *potential* $\pi \in \mathbb{R}^N$ such that $\forall (i,j) \in A \quad \theta_{ij} = \pi_j - \pi_i$.

Minimum cost tension problem (MCT) is finding a tension θ satisfying lower ($t_{ij} \in \mathbb{R} \cup \{-\infty\}$) and upper ($T_{ij} \in \mathbb{R} \cup \{+\infty\}$) bounds on each arc such that $\sum_{a_{ij} \in A} c_{ij}\theta_{ij}$ is minimum. In *maximum tension problem (MaxT)*, the graph G contains 2 special nodes, s and t, and an arc $a_{st} \in A$ between these two nodes with bounds $(t_{st}, T_{st}) = (-\infty, \infty)$. The maximum tension problem is finding the maximum tension on arc $a_{st} \in A$ such that the tensions on all arcs satisfy the upper and lower bounds. In this study we assume that both problems are feasible and have finite optimal solutions. Our aim is to analyze their inverse versions.

Given a feasible tension $\hat{\theta}$ to an instance of a MCT, the *cost inverse minimum cost tension problem* (IMCT_c) is perturbing the cost vector from c to $\hat{c}$ in a way that $\hat{\theta}$ will become the optimum tension for the minimum cost tension problem with the perturbed cost vector ($\text{MCT}(\hat{c})$) while the perturbation $\|c - \hat{c}\|$ is minimized according to some norm. On the other hand, in *inverse maximum tension problem* (IMaxT) we modify the bound vectors from T to $\hat{T}$ and/or from t to $\hat{t}$ such that $\hat{\theta}_{st}$ will become the maximum tension with the perturbed bound vectors. We exploit L_1 and L_∞ norms to measure the parameter modifications.

The main results of this study are as follows. Inverse minimum cost tension problem under unit weight rectilinear norm is equivalent to finding a minimum cost collection of arc-disjoint residual cuts in G. Let μ^* denote the mean cost of a minimum mean residual cut in G w.r.t. $\hat{\theta}$. The optimal objective function value for inverse minimum cost tension problem under L_∞ norm is $max(0, -\mu^*)$. The solution to the inverse maximum tension problem under L_1 norm with a positive weight function w can be found by solving a maximum tension problem in graph G.

Monotropic programming deals with optimization problems that minimize a separable convex function subject to linear constraints. Several optimization problems such as linear and quadratic programs, network flows and tensions are special cases of monotropic programs. In this study, we analyze the inverse problems of monotropic programs with seperable linear cost functions and show that the combinatorial solutions of Ahuja and Orlin [1] can be extended for these problems.

Literatur

[1] R.K Ahuja and J.B. Orlin. Combinatorial algorithms of inverse network flow problems. *Networks*, 40:181–187, 2002.

[2] R.T. Rockafellar. *Network Flows and Monotropic Optimization*. John Wiley and Sons, 1984.