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Cash Recycling Systems: Prediction and Optimization

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We predict the future turnovers of Cash Recycling System (CRS) by simple and efficient time series methods using the past data. Based on this prediction, we present an Mixed Integer Programming approach to find optimal replenishment schedules in reasonable running-time.

1 Introduction

Cash Recycling Systems (CRS) are Cash Points (CP) that dispense money but are also able to accept deposited money (bank notes and coins). This new type of CP has been introduced recently by financial institutions. Since the data basis for CPs operated by these financial institutions is rather inhomogeneous and often quite erroneous and the number of reliable sources for CRS is very small, we base our analysis on the time series (x) of daily turnovers. Thus we do not have to take into account the number of clients and refillings. On the other hand we can find good predictions from very few data.

2 Prediction

The left picture of Figure 1 shows 30 days of the cumulated turnovers of a CRS without special season. This time series has no obvious pattern, so we concentrate on the time series of outgoing and incoming cash flows (Figure 1, right picture, blue curve). We can estimate the mean (μ) and the weekly (w) and monthly (m) periodic components of both time series using

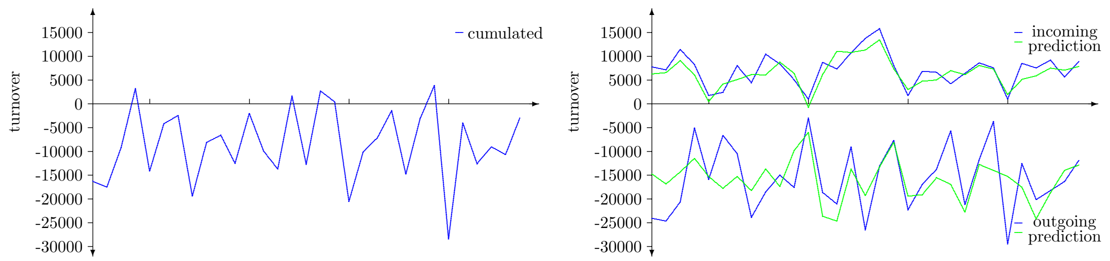

Fig. 1 Cumulated Turnovers (left), Turnovers and Predictions (right)

standard time series methods. For details see, e.g., [1]. On the other hand the cash flow is influenced by the customers special behaviour in the middle and the end of the month. These effects have to be corrected by the terms U and M, which are determined by suitable heuristics. Thus the prediction of the time series of the daily turnovers reads as follows:

$$\widehat{x} = \mu + w + m + U + M \tag{1}$$

Due to the random withdrawals and deposits of the customers it is not possible to derive exact forecasts. Therefore we have to take into account the uncertainty (see the right picture of Figure 1, where the prediction (green curve) is not exact) of the future transactions by adjusting the prediction with an adequate security loading. The security loading can be assumed to be proportional to the standard deviation (σ) of the prediction error, which is defined in equation (2):

$$\text{error} = x - \widehat{x} \tag{2}$$

This is motivated by Figure 2, and further analysis indicates that the error is close to white noise. Note that the standard deviation of the prediction error of the incoming money is three times as large as for the dispensed money and that thus we cannot predict the days when high amounts of money are paid in. The effect of the security loading can be seen in Figure 3.

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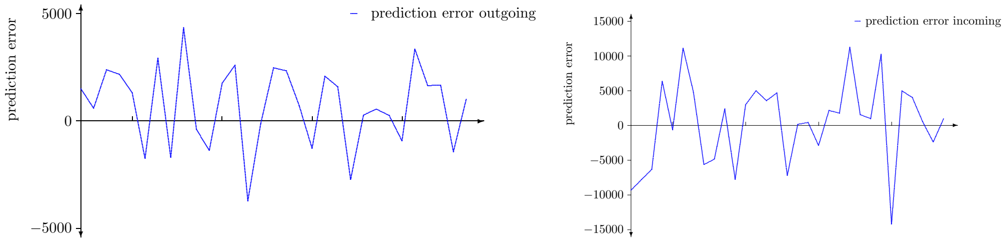

Fig. 2 Prediction error for outgoing (left) and incoming (right) cash flows

3 Optimization

An optimal replenishment schedule can be found using Mixed Integer Programming. Here the aim is to minimize all arising costs such as transport and handling costs (δ), interest costs (λ), and penalty costs for empty or full CPs (ρ). The core of the optimization model reads as follows:

Decision Variables

w_t^i	$\in \{0,1\}$	with $w_t^i = 1$ iff CP i is supplied on day t
$s_t^{i,d}$	$\in \mathbb{Z}_0^+$	stock of denomination d in CP i on day t
$y_t^{i,d}$	$\in \{0,1\}$	with $y_t^{i,d} = 0$ iff denomination d is not available in CP i on day t

Objective Function

$$\text{Minimize} \underbrace{\sum_{t,i,d} \lambda_t d s_t^{i,d}}_{\text{interests}} + \underbrace{\sum_{t,i} \delta_t^i w_t^i}_{\text{transport \& handling}} + \underbrace{\sum_{t,i} \rho_t^i (1 - y_t^i)}_{\text{penalty}}$$

When minimizing the above formula, we have to consider a number of constraints, as for instance the number of deliveries per day is limited, the stock of a CP is restricted by technical and actuarial bounds etc. Furthermore, the variables have to be consistent, for instance $y_t^{i,d} = 0$ iff $s_t^{i,d} = 0$.
Since problems of realistic size contain up to 1000 CPs and more, yielding models with up to 200,000 variables and 250,000 constraints, we have to decrease the problem's complexity by grouping CPs and using parallel computations. Moreover, we compute the replenishment volume in advance, so that it only remains to find the optimal replenishment dates.
For details see [2].

4 Computational Results

Table 4 shows running times of specific scenarios using ILOG Cplex [3] with default settings on Pentium III 1000MHz.

#CPs	#Days	Gap	Running Time
30	15	None	0.5 min
100	15	None	28 min
100	15	10%	2.5 min
400	15	10%	15 min
800	15	10%	330 min
800	10	10%	12 min

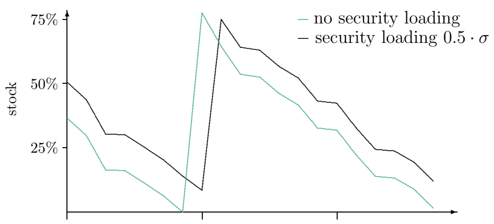

Fig. 3 Typical replenishment schedule

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[2] U. Günther, A. Martin, K. Ritter, T. Wagner, Optimal Loading Strategies for large systems of Cash Points. Preprint, TU Darmstadt, Fachbereich Mathematik, in progress.
[3] ILOG CPLEX Division, 889 Alder Avenue, Suite 200, Incline Village, NV 89451, USA. Information available at URL http://www.cplex.com